Validation of a simplified method in viscoelastic continuum damage (VECD) model developed for flexural mode of loading

Farshad Haddadi, Mahmud Ameri, Mohammad H. Mirabimoghadam, Hamid R. Amiri Hosseini

Department of Civil Engineering, Iran University of Science & Technology, Tehran, Iran
Transportation and Safety Iran University of Science & Technology, Tehran, Iran
Civil Engineering Department, Nikbakhht College of Engineering, Sistan and Baluchestan University, Zahedan, Iran

Highlights
- Simplified VECD model using peak-to-peak values of stress and strain is applicable in flexural mode of loading.
- Material constant ($a$) has inverse relationship with slope of the central portion of the relaxation modulus curve obtained from the flexural dynamic test.
- There is only limited range of initial pseudostiffness ($I$) for which the errors in calculation of damage parameters could be neglected.

Abstract
Viscoelastic continuum damage (VECD) is a powerful method to describe fatigue behavior of asphalt concrete mixtures by implementing the fundamental mechanistic principles. In this study, peak-to-peak values of stress and strain are used to simplify the calculation of damage parameters in VECD model using cyclic flexural fatigue test. Fatigue tests have been conducted over a range of strain and frequency levels. It was observed that excluding the samples with initial pseudostiffness ($I$) less than 0.74, other specimens’ characteristic curve overlaps on a unique curve for any set of loading. Also, the results showed that there is an inverse relationship between the material constant ($a$) and the slope of the central portion of the relaxation modulus curve obtained from a flexural dynamic test using 4-point bending beam apparatus.

1. Introduction
Fatigue cracking is a major type of structural distress which often occurs at the surface of the asphalt pavements. Due to increase concern with pavement cracking, the importance of introducing a reliable method for measuring fatigue parameters of asphalt mixtures has been widely recognized. For many years, researchers attempted to introduce a reliable method for predicting fatigue performance of asphalt mixture. Although some advancement has been achieved but there is no unanimously agreed approach to describe fatigue behavior and/or to investigate fatigue characteristics of asphalt concrete mixtures. To get better insight into the fatigue phenomena of asphalt mixtures a proper mechanistic model is required which could describe different levels of initiation, coalescing and propagation of crack under a repeated mode of loading. However, many asphalt agencies still use empirical techniques to estimate the fatigue life of asphalt pavement through their design procedure. These techniques usually characterize fatigue behavior of an asphalt mixture by plotting the maximum tensile strain or stress against number of load cycles to failure [1]. Such an approach not only could be time consuming but also it fails to account for the most fundamental aspects of the asphalt concrete such as its viscoelastic nature. On the other hand, the Continuum Damage Mechanic (CDM) is gaining momentum among most of the asphalt communities in recent years. It offers more fundamental explanation of damage than traditional fatigue approaches. Several studies have been undertaken to approve the applicability of CDM approach for evaluation of crack growth in the asphalt pavements based on Schapery’s correspondence principle and the Work Potential Theory (WPT) [2–7].

Schapery adopted the well-known power law crack growth equation for viscoelastic materials, and developed his viewpoint mathematically through Eq. (1) to represent the damage evolution process of materials in viscoelastic media through the correspondence principle [8,9]:

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where, \( S \) is internal state variable (or damage parameter), \( W^R \) is pseudostrain energy density and \( \alpha \) is the material constant. Some researchers used trial and error procedure to defined the \( \alpha \) value as a constant which could generate the unique damage curve for various load amplitudes [4,7]. Other researchers attempted to relate material constant to the slope (\( m \)) of the central portion of relaxation modulus master curve [10,11]. They also observed that if the material’s fracture energy and failure stress are constant, then the \( \alpha \) value could be defined as 1 + 1/\( m \). On the other hand, if the fracture process size and fracture energy are constant, then the \( \alpha \) value could be defined as 1/\( m \). Swamy et al. used the rigorous calculation of pseudostrain (Eq. (2)) for calculating damage parameters and reported 1 + 1/\( m \) as the proper definition of \( \alpha \) at controlled strain amplitude in flexural dynamic test [12]:

\[
\frac{ds}{dt} = \left( -\frac{\partial W^R}{\partial \varepsilon} \right)^2
\]

(1)

where, \( E_s \) is the reference modulus, \( E(t) \) is the relaxation modulus, \( \varepsilon \) is the physical strain, \( t \) the elapsed time between the beginning time of loading and the time of interest and \( \tau \) is the time when the loading began.

Although the CDM has many advantages over empirical methods, application of this approach to a large number of cyclic load data is sometimes problematic issue since using the hereditary integral (Eq. (2)) through entire loading history demands a huge computation memory. Furthermore, noises of the output data could adversely affect the results of the experiment and cause considerable errors [6]. Some studies have been conducted to overcome these problems. Christensen and Bonaquist presented a number of practical applications of the modified continuum damage model based on works of Kim et al. [13] Kutay et al. used peak-to-peak values of stress and strain to construct damage curves in cyclic uniaxial laboratory fatigue tests and proved its sufficient accuracy [5]. Underwood et al. improved earlier versions of simplified approaches and developed a more accurate method [6].

Most of the aforementioned studies were based on the results of uniaxial fatigue test on cylindrical samples but very few have applied VECD approach into the flexural mode of loading. In this study, the 4-point bending beam fatigue test is employed to investigate the possibility of using peak-to-peak strain and stress data values (instead of entire loading history) so as to simplify damage calculation in VECD model. In addition, the beam fatigue apparatus was utilized in order to construct the relaxation modulus master curve by conducting a dynamic flexural test.

2. Laboratory test program

2.1. Materials

The aggregate used in this study is limestone with a maximum nominal aggregate size of 19-mm obtained from the stockpiles of a local asphalt plant. The gradations and characteristics of aggregate are presented in Tables 1 and 2, respectively. Two types of base binder (unmodified) with different Performance Grade (PG) status were used in this study. Also, 4% ethylene vinyl acetate (EVA) copolymer by weight of bitumen was used to modify the two base binders, utilizing laboratory high shear mixer. EVA used in this study had vinyl acetate content of 18% by mass, specific gravity of 0.958 g/cm³, melt indices (MI) of 2.5 g/10 min and melting point of 85 °C. P1 and P2 will denote the mixtures made with modified PG64-16 and PG58-22 base binders, respectively.

2.2. Sample preparation

The optimum bitumen content of the mixtures determined using standard Marshall mix design procedure (ASTM D1559) [20]. All data collected in this study including LVE characteristics and fatigue parameters of the mixtures were obtained from testing of asphalt concrete beam specimens with dimensions of 5 × 6 × 39 cm. The samples were originally fabricated with dimensions of 5 × 30 × 40 cm using wheel track compactor and then were sawed to the desired dimensions. Table 3 summarizes mixtures type, bitumen content, bitumen grade and air void of the samples tested in this study.

2.3. Dynamic modulus and fatigue test

Traditional four-point bending beam fatigue testing apparatus was utilized to characterize LVE property and fatigue behavior of asphalt mixtures. The specimens were subjected to 70 microstrain (\( \mu \varepsilon \)) sinusoidal cyclic loading at frequencies of 0.1, 0.5, 1.0, 5.0, 10.0, 15 and 20 Hz; and temperatures of −10 °C to 40 °C in 10 °C increments to obtain their dynamic modulus (\( E' \)) and phase angle. To allow for specimen recovery, a pause period of four minute was applied before implementing each level of frequency. In the next stage, the beam fatigue tests were conducted in accordance with AASHTO-T 321 [21] at 20 °C using 400, 700 and 900 microstrain and sinusoidal loading at 7, 10 and 13 Hz. Three replicates were used for each test.

3. Results and discussion

3.1. LVE characterization

Time–temperature superposition principle was utilized to construct the dynamic storage modulus master curve at a reference
temperature of 20 °C for each type of the mixtures as shown in Fig. 1. To carry out the task, the dynamic modulus obtained from various temperature and frequencies were shifted along with frequency values until they fitted into a sigmoidal function represented by Eq. (3):

$$\log |E'| = \delta + \frac{\alpha}{1 + \frac{T}{f_r}}$$

where \(f_r\) is reduced frequency at reference temperature, Hz; \(\delta\) is minimum value of \(E'\); \(\delta + \alpha\) is maximum value of \(E'\); and \(\beta, \gamma\) are parameters, describing the shape of the sigmoidal function. Eq. (4) was used to determine the corresponding shift factors:

$$a(T) = \frac{f_r}{f}$$

where \(a(T)\) is the shift factor as a function of temperature; \(f\) is frequency at desired temperature, Hz and \(T\) is desired temperature, °C.

For more precision, a second order polynomial expression was used to relate the logarithm of the shift factor to temperature [22] which is described by Eq. (5):

$$\log a(T) = at^2 + bt + c$$

where \(a, b\) and \(c\) are coefficients of second order polynomial function.

Finally, a least square technique was used to simultaneously optimize values of \(\alpha, \delta, \beta, \mu\) in Eq. (3) and \(a, b, c\) in Eq. (5), so that the difference between measured dynamic modulus and fitted values of the sigmoidal function is minimized and the shift factors at the reference temperature (which in this experiment is 20 °C) is equal to unity. Computations of all above process were conducted with the aid of Microsoft Excel using Solver function. The constant values of \(a, b, c, \alpha, \delta, \beta, \mu\) parameters for each type of mixtures are presented in Table 4.

The master curve parameters and shift factors obtained from the dynamic storage modulus master curve were then used to construct the relaxation modulus master curves using approximate interconversion method proposed by Park and Schapery [23]. The relaxation modulus master curves of the mixtures at 20 °C are shown in Fig. 2. For more precision, a second order polynomial expression was used to relate the logarithm of the shift factor to temperature [22] which is described by Eq. (5):

$$\log a(T) = at^2 + bt + c$$

where \(a, b\) and \(c\) are coefficients of second order polynomial function.

Finally, a least square technique was used to simultaneously optimize values of \(\alpha, \delta, \beta, \mu\) in Eq. (3) and \(a, b, c\) in Eq. (5), so that the difference between measured dynamic modulus and fitted values of the sigmoidal function is minimized and the shift factors at the reference temperature (which in this experiment is 20 °C) is equal to unity. Computations of all above process were conducted with the aid of Microsoft Excel using Solver function. The constant values of \(a, b, c, \alpha, \delta, \beta, \mu\) parameters for each type of mixtures are presented in Table 4.

The master curve parameters and shift factors obtained from the dynamic storage modulus master curve were then used to construct the relaxation modulus master curves using approximate interconversion method proposed by Park and Schapery [23].

### Table 4: Master curve and shift parameters.

<table>
<thead>
<tr>
<th>Mixture</th>
<th>3.6577</th>
<th>3.2495</th>
<th>0.3413</th>
<th>0.3759</th>
<th>-0.0002</th>
<th>-0.056</th>
<th>4.5378</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>4.0485</td>
<td>2.6735</td>
<td>0.0539</td>
<td>0.5102</td>
<td>0.00002</td>
<td>-0.074</td>
<td>4.9427</td>
</tr>
<tr>
<td>C2</td>
<td>4.6037</td>
<td>2.0609</td>
<td>0.1253</td>
<td>0.5486</td>
<td>0.00006</td>
<td>-0.079</td>
<td>5.119</td>
</tr>
<tr>
<td>P1</td>
<td>4.5412</td>
<td>2.0768</td>
<td>0.1905</td>
<td>0.5866</td>
<td>-0.0005</td>
<td>-0.016</td>
<td>3.3594</td>
</tr>
<tr>
<td>P2</td>
<td>4.5412</td>
<td>2.0768</td>
<td>0.1905</td>
<td>0.5866</td>
<td>-0.0005</td>
<td>-0.016</td>
<td>3.3594</td>
</tr>
</tbody>
</table>

Relaxation modulus master curves for all four types of the mixtures are presented in Fig. 2.

### 3.2. Damage characterization

Since distribution of stress and strain within the prismatic specimen under flexural mode of loading is not uniform [24], for simplicity, only stress and strain value in the upper or lower portion of the bending beam’s cross-section were used to develop continuum damage model. The on-specimen displacement measurement at the middle of the beam was utilized to calculate the maximum strain and stress at each loading cycle using Eqs. (6) and (7), respectively:

$$\sigma = \frac{0.357P}{bh^2}$$

$$\varepsilon = \frac{12a\delta}{3L^2 - 4a^2}$$

where \(b\) is the average specimen width, \(h\) is the average specimen height, \(a\) is the distance between inside clamps, \(L\) is the distance between outside clamps, \(\delta\) is the deflection at center of beam at a certain time and \(P\) is the load applied by actuator.

Then, the concept of pseudostrain was used to convert constitutive equations in time dependent viscoelastic media to a simple linearly elastic case. The physical implication of pseudostrain is linear viscoelastic stress for a given strain history. When material is tested under few cyclic loading within the 50–75 με range (similar to Section 3.1), the induced damage is negligible [6], hence pseudostrain is equal to stress at every cycle; but when relatively high strain amplitude is used, material will gradually lose integrity due to formation of microcracks, and hence the slope of stress vs. pseudostrain curve, which is called pseudostiffness (C), attenuates toward zero. In this study, behavior within each cycle for stress vs. pseudostrain curve is assumed to be linear and tests were considered to be in steady-state condition. Thus, computation of pseudostrain was possible through the simplified Kutey’s approach [5]. In Kutey’s simplified approach, the rigorous and time consuming computation for the solution of hereditary integral given in Eq. (2) is eliminated and Eq. (8) is used for calculation of peak-to-peak pseudostrain (\(\varepsilon_{pp}\)).

$$\frac{\varepsilon_{pp}}{E_{v50} = E_{v50} \times \varepsilon_{pp}}$$

where \(E_{v50}\) is the linear viscoelastic dynamic modulus which is specific to the temperature and frequency of the fatigue test under consideration and is obtained from dynamic modulus master curve. Table 6 shows \(E_{v50}\) values corresponding to loading frequencies of 7,
10, 13 Hz at temperature of 20 °C. Because the pseudostrain calculation in this method is based on peak-to-peak values, it assumes that both the tension and compression are participating in the damage growth. However, Daniel showed that damage can only accumulate during the tensile portion of each cycle [4]. This assumption is also corroborated by the works of Underwood et al. and Mello et al. [6,24]. According to this assumption, in this study, only the tensile half of each cycle is used to determine pseudostrain.

By proper calibration and adjustment of the on-specimen Linear Variable Differential Transformer (LVDT) and the actuator gauge, duration of tensile portion of each cycle would be equal to compressive portion; therefore, tensile pseudostrain ($\varepsilon_{pp}^t$) can be simply defined as half of the peak-to-peak pseudostrain as follows:

$$\varepsilon_{pp}^t = \frac{\varepsilon_{pp}}{2} \tag{9}$$

To make it more clear, a common set of variables that are used in this study are summarized in Fig. 3 at an arbitrary loading cycle in strain controlled mode of loading.

To accounts for specimen to specimen variability, pseudostiffness is normalized using initial pseudo stiffness ($l$) with aid of Eq. (10):

$$C_i = \frac{\sigma_{ro}}{I \cdot (\varepsilon_{pp}^t)} \tag{10}$$

where $C_i$ is normalized pseudostiffness and

$$I = \frac{(\sigma_{ro})_1}{(\varepsilon_{pp}^t)_1} \tag{11}$$

where $\sigma_{ro}$ and $\varepsilon_{pp}^t$ are stress and pseudostrain at the end of the first load cycle, respectively. In this study $l$ values varied over a range of 0.45–1.05. As presented in Table 5, lower levels of strain/frequency could lead to $l$ value closer to unity. This can be due to the fact that small strain and frequency amplitude have minor effect on the integrity of the specimen during the first load cycle [6], hence minor loss of integrity within first cycle would have negligible effect on dynamic modulus value and thus at the end of the first cycle, where $l$ value is calculated, the dynamic modulus would be close to its linear state ($E_{LVE}$) value. Therefore, since the test is in strain controlled mode and also according to Eqs. (8) and (11), smaller strain and frequency amplitude, results in $(\sigma_{ro})_1$ values closer to $(\varepsilon_{pp}^t)_1$ and thus $l$ value would approaches to unity.

Finally, the damage parameter was determined using Eq. (12):

$$S = \sum_{i=1}^{N} \left[ \frac{I}{2} (C_{i+1} - C_{1}) (\varepsilon_{pp}^t)_1 \right]^{\frac{1}{m}} = \left( \frac{\Delta N}{\frac{2}{f}} \right)^{\frac{1}{m}} \tag{12}$$

where $x$ is the material constant; $f$ is load frequency, Hz; and $\Delta N$ is the number of the loading cycles during which damage ($S$) is calculated.

Since in most studies, the $x$ value is defined based on m value that are investigated through uniaxial mode of loading, in this study the interactions between $x$ and m is further investigated through flexural dynamic loading. Formerly researchers have shown that the characteristic curve is material propriety and captures the fundamental properties of material [4]. Based on this fact, iterative method was used to find an $x$ value that yields the best overlaps of the characteristic curves for a certain type of mixture, regardless of the level of strain and frequency of the test. Results indicate that material constant ($x$) and m parameter are inversely proportional, particularly for the set of test data obtained in this study. Table 6 shows the Alpha and $m$ values for each type of the mixtures.

Characteristic curves are determined by plotting pseudo stiffness against damage parameters. The generalized exponential model (Eq. (13)) rather than generalized power model was used to fit the damage characteristic curve because it yielded a better fit to the data of this research study:

$$C_i = \exp(k_1 \times S^2) \tag{13}$$

where, $C_i$ is normalized pseudo stiffness, $S$ is damage parameter and $k_i$ is regression coefficient.

As it can be observed in Fig. 4a, all of the specimens made of C2 mixture, as an example, loaded under different strain levels and

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**Table 5**

Values of $l$ parameter for each type of the mixtures under different levels of strain and frequency at 20 °C.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>7 Hz</th>
<th>10 Hz</th>
<th>13 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mixture type</td>
<td>Strain level</td>
<td>400 µε</td>
<td>700 µε</td>
</tr>
<tr>
<td>C1</td>
<td>0.95</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>C2</td>
<td>1.05</td>
<td>0.92</td>
<td>0.93</td>
</tr>
<tr>
<td>P1</td>
<td>0.85</td>
<td>0.79</td>
<td>0.75</td>
</tr>
<tr>
<td>P2</td>
<td>0.91</td>
<td>0.82</td>
<td>0.78</td>
</tr>
</tbody>
</table>

---

**Table 6**

Values of the best fit coefficients of the model characteristic curve ($k_1, k_2, m$) obtained from relaxation modulus curve, Alpha value obtained from iterative method and $E_{LV}$.

<table>
<thead>
<tr>
<th>Type of mixture</th>
<th>$m$</th>
<th>$x$</th>
<th>$k_1$</th>
<th>$k_2$</th>
<th>$E_{LV}$ at 20 °C (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>f = 7 Hz</td>
<td>f = 10 Hz</td>
<td>f = 13 Hz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C1</td>
<td>0.3598</td>
<td>2.8</td>
<td>0.000122</td>
<td>0.47931</td>
<td>6180</td>
</tr>
<tr>
<td>C2</td>
<td>0.3448</td>
<td>2.9</td>
<td>0.001158</td>
<td>0.4792</td>
<td>4579</td>
</tr>
<tr>
<td>P1</td>
<td>0.2770</td>
<td>3.6</td>
<td>0.000011</td>
<td>0.6839</td>
<td>6636</td>
</tr>
<tr>
<td>P2</td>
<td>0.2675</td>
<td>3.7</td>
<td>0.000066</td>
<td>0.5232</td>
<td>6267</td>
</tr>
</tbody>
</table>

---

**Fig. 3.** Diagrammatic outlook of tensile stress ($\sigma_{ro}$), peak-to-peak stress ($\sigma_{pp}$), tensile pseudostrain ($\varepsilon_{pp}^t$) and peak-to-peak pseudostrain ($\varepsilon_{pp}^t$) and pseudostiffness ($C_i$) in the simplified approach.
4. Conclusion

The prime objective of this research study was to employ the 4-point bending beam apparatus to investigate possibility of using peak-to-peak stress/strain values of each cycle (in oppose to entire loading history), in order to determine damage parameters of VECD model. It was found that for specimens with I values less than 0.74, there exist a particular value for which all damage curves overlap on a unique curve. The value in damage equation was estimated as $1/m$ in which $m$ is the central slope of the relaxation modulus vs. time curve, plotted in logarithmic space. The uniqueness of the damage characteristic curve is consistent with the findings of other researchers that used uniaxial mode of loading.

Tentative findings of this study indicate that the more I value is closer to unity, the better superposition of damage curves could be obtained for a particular value of $\alpha$. This could be largely attributed to the significant damage growth during the first cycle specifically at the higher levels of strain and/or frequency; in this regard, peak-to-peak values could not be a proper representative of the material state at the first cycle.

**References**


frequency amplitudes (except those subjected to 900 and 700 $\mu$m at the frequency of 13 Hz), are fitted by a single characteristic curve using $\alpha = 1/m$. Recent definition of $\alpha$ parameter was also consistent for other types of the mixtures considered in this study. However, it was observed that those specimens for which their I value is less than 0.74 do not overlap on a unique curve for any particular $\alpha$ value.

**Table 6** presents the values of $k_1$ and $k_2$ of exponential function that best fit the data of each type of mixtures and **Fig. 5** represent their corresponding damage characteristic curves, so as to make them visually comparable.


